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FILTERING AND CONTROL PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

A. V. Balakrishnan

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FILTERING AND CONTROL PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

A. V. Balakrishnan †

1. Introduction

Many problems of filtering and control involving partial differential equations can be described abstractly in a manner similar to the ordinary differential equation case by invoking the theory of semigroups of operators [1]. We have thus a 'state equation' of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + Fn(t)$$

where A is the infinitesimal generator of a strongly continuous semigroup, u(.) is the 'input', n(.) is the noise and B and F are bounded linear operators. The 'observation' y(t) has the form

$$y(t) = Cx(t) + Gn(t)$$

where C and G are bounded linear operators. An optimal control and filtering theory for such a system has been developed in [1]. However, the extension to the case where the operators B, F and C are not bounded turns out to be quite significant. In fact, they need to be not only unbounded; but also uncloseable. We shall treat two typical cases illustrating such extensions. The first problem (B unbounded) occurs when the control is on the boundary (of which there is no finite dimensional analogue). The second problem, of C unbounded, uncloseable, occurs whenever we wish to consider "point-wise" observations (in the domain or on the boundary).

We begin with the boundary control problem.

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2. Boundary Control

In this section we shall consider the linear (deterministic)+ quadratic regulator problem for a class of diffusion equations when the control is on the boundary. Lions [3] has treated such a problem but he requires that the control be in $H^{1/2}$ on the boundary (in particular requiring C_0^∞ boundaries). This restriction seems somewhat artificial and in any case here we require only that the controls be in L_2 on the boundary. Also we exploit semigroup theory. The main tool is the construction of a generalized solution for boundary inputs (see [1] for an elementary exposition).

Let $\mathcal{D}$ denote a bounded domain with boundary Γ in real Euclidean space R^n. Points in the space will be denoted by ξ, with components ξ_i. Let τ denote a second order strongly elliptic operator:

$$\tau f = \sum_1^m \sum_1^m a_{ij}(\xi) \frac{\partial^2 f}{\partial \xi_i \, \partial \xi_j} + \sum_1^m a_i(\xi) \frac{\partial f}{\partial \xi_i} \quad \text{in } \mathcal{D} \tag{2.1}$$

where the coefficients are continuous in the closure of $\mathcal{D}$.

We consider the control problem for the equation:

$$\left.\begin{aligned} &\frac{\partial f}{\partial f} = \tau f \quad 0 < t, \quad \xi \text{ in } \mathcal{D} \\ &f(t,.)\big|_\Gamma = u(t,.) \\ &\quad f(0,\xi) \text{ given} \end{aligned}\right\} \tag{2.2}$$

where $u(t,.)$ is in $L_2(\Gamma)$ for each t. Before we specify the cost function, we shall develop the semigroup theoretic approach to the solution of (2.2). For this purpose we first consider the special case of controls $u(t,.)$ which are strongly continuously differentiable in $0 \leq t < \infty$.

+For a treatment of stochastic boundary control see [2].

Let A denote the 'zero boundary value' restriction of τ, more specifically, let A denote the smallest closed extension in $L_2(\mathscr{D})$ of τ restricted to $C_0^\infty(\mathscr{D})$. Assume that τ is strongly elliptic so that the quadratic form:

$$\sum_1^m \sum_1^m a_{ij}(\xi)\, X_i X_j > \beta \sum_1^m X_i^2, \ \beta > 0, \text{ for all } \xi \text{ in } \mathscr{D}.$$

Then A generates a strongly continuous semigroup over $L_2(\mathscr{D})$, analytic in a sector in the right-half plane. Denote the semigroup S(t). We assume next that the boundary Γ is such that the Dirichlet problem

$$\tau f = 0 \text{ in}$$

$$f\Big|_\Gamma = g$$

where g is in $L_2(\Gamma)$ has a unique solution given by

$$f = Dg$$

where, furthermore, D is a linear bounded transformation mapping $L_2(\Gamma)$ into $L_2(\Omega)$.

Then following a technique due to Fattorini [4] we can express the solution of [2.2] in abstract form as [see [1]]:

$$x(t) = S(t)(x(o)-Du(o))+\int_0^t S(t-s)\, D\dot{u}(s)ds \qquad (2.3)$$

where x(o) is the initial function (assumed to be in $L_2(\mathscr{D})$). For each t, x(t) describes the solution as an element of $L_2(D)$ and is strongly continuous in t. For x(o) in the domain of A, the solution satisfies (2.2) and moreover

$$||x(t) - x(o)|| \to 0, \quad \text{as} \quad t \to 0+$$

and of course x(t) 'assumes' the boundary value u(t). See [1] for the uniqueness of the solution (2.3).

While (2.3) is an acceptable solution, it has the disadvantage that the derivative of u(t) appears and we want to avoid this. It is here that we introduce our notion of a 'generalized' solution, sacrificing the "pointwise" interpretation of (2.2).

The Generalized Solution of the Boundary Input Problem

We exploit now the fact that the semigroup S(t) is analytic. From this it follows [see [5] for a proof] that for any u(.) in

$$W_b = L_2[[0,T]; L_2(\Gamma)]$$

we have that

$$\int_0^t S(t-\sigma)\, Du(\sigma)d\sigma$$

belongs to the domain of A a.e. in [0,T] and that a.e:

$$A\int_0^t S(t-\sigma)\, Du(\sigma)d\sigma = \int_0^t AS(t-\sigma)\, Du(\sigma)d\sigma = g(t) \quad 0 < t < T$$

where g(t) belongs to

$$W_s = L_2[0,T; L_2(\mathscr{D})]$$

Moreover

$$Lu = g; \; g(t) = -\int_0^t AS(t-\sigma)\, Du(\sigma)d\sigma \quad \text{a.e} \tag{2.4}$$

defines a linear bounded transformation mapping W_b into W_s. Integrating by parts in (2.3) and exploiting the fact that L is bounded, we can obtain the 'generalized solution' which is now valid for any u(.) in W_b, as:

$$x(t) = S(t)\, x(0) - \int_0^t AS(t-s)\, Du(s)ds \quad \text{a.e } 0 < t \tag{2.5}$$

and here the a.e. qualification is crucial. For the details, we refer once again to [1]. Note that A commutes with S(t) on the domain of A and hence it is <u>as if</u> we have

$$AS(t-s)\, Du(s) \approx S(t-s)\, Bu(s)$$

where B roughly speaking is "AD", although of course, A is simply not defined on the range of D!

We can now specify the optimization problem. Given Q, linear bounded, mapping $L_2(\mathscr{D})$ into another Hilbert space, it is required to minimise:

$$q(u) = \int_0^T [Qx(t),\, Qx(t)]dt + \int_0^T [u(t), u(t)]dt \tag{2.6}$$

for u(.) in W_b, with x(.) given by (2.5). Note that we can recast (2.5) in the form:

$$\left.\begin{aligned} x(t) &= S(t)x(0) - A\, y(t) \text{ a.e} \\ \dot{y}(t) &= A\, y(t) + Du(t) \text{ a.e} \\ y(0) &= 0 \end{aligned}\right\} \tag{2.7}$$

As the first step towards the characterization of the optimal control, let us calculate first L^*, the adjoint of L since it will play a key role. We have

$$L^*x = u;\ u(t) = \int_t^T - D^* (AS(s-t))^* x(s)ds \quad \text{a.e.} \tag{2.8}$$

Now for $t > 0$

$$D^* (AS(t))^* = D^* A^* S(t)^*$$

Let us set

$$C = D^* A^*$$

Then C is clearly defined on the domain of A^* (and hence also in the range of $S(t)^*$ for $t > 0$) but C is not closeable. In fact in the simple case where τ is actually the Laplacian, we can see (by Green's theorem) that C reduces to the normal derivative on the boundary Γ and hence is clearly not closeable. Indeed if C were closeable, its adjoint would be AD which is of course not defined except on the zero element.

Of course

$$C\, S(t)^*$$

is linear bounded for each $t > 0$ and

$$\int_t^T CS(s-t)^* Q^*Q\, x(s)ds \ \text{a.e} \quad 0 < t < T$$

defines an element of W_b for $x(.)$ in W_s.

By the same kind of theory as in the finite dimensional case, we can establish the existence of unique optimal control $u_0(.)$ given by (see [1] for instance):

$$u_0 = -(QL)^* (Q\,Lu_0 + Q\omega) = -(QL)^*\,Q\,x_0(\cdot)$$

where ω is the element in W_s defined by

$$S(t)\,x(0) \quad 0 < t < T$$

We can express $u_0(.)$ as:

$$u_0(t) = -\,C\,z(t)$$

where $z(t)$ is the solution of

$$\dot{z}(t) + A^*\,z(t) = -Q^*Q\;x_0(t)\;;\; z(T) = 0\ldots \tag{2.9}$$

By analogy with the finite dimensional case, we expect that we must be able to express $z(t)$ as

$$z(t) = P(t)\,x_0(t)$$

where $P(t)$ is a linear bounded self-adjoint transformation mapping $L_2(\mathcal{D})$ into itself, satisfying, for x,y in domain of A, a.e. in t:

$$\left.\begin{array}{l} [\dot{P}(t)x,y] + [P(t)Ay,x] + [P(t)x,Ay] \\ \;-\;[CP(t)x,CP(t)y] + (Qx,Qy] = 0 \\ P(T) = 0 \end{array}\right\} \tag{2.10}$$

Note this implies in particular that for any x in $L_2(\mathcal{D})$,

$$P(t)x \in \text{ Domain of } C, \text{ a.e. } 0 < t < T$$

for which it is enough if for $x \in (A)$,

$$P(t) \in \text{ Domain of } A^*, \text{ a.e. } 0 < t < T$$

Assume now that we have found such a $P(t)$. Then we can show that $P(t)\, x_o(t)$ satisfies (2.9). We have

$$\frac{d}{dt}\, [P(t)\, x_o(t), y]$$

$$= \frac{d}{dt}\, [S(t)\, x(0) + Ay_o(t),\, P(t)y]$$

where

$$\dot{y}_o(t) = Ay_o(t) + D\, u_o(t);\; y(0) = 0$$

The main calculation is that

$$[P(t)\, \dot{x}_o(t), y] = [P(t)\, AS(t)\, x(0), y] - [A\, y(t) + D\, u(t),\, A^*P(t)y]$$

$$= [x_o(t),\, A^*\, P(t)y] - [u(t),\, CP(t)y]$$

$$= [x_o(t),\, A^*\, P(t)y] + [CP(t)\, x_o(t),\, CP(t)y]$$

A little arithmetic shows that

$$\dot{P}(t)\, x_o(t) \tag{2.11}$$

does satisfy (2.9), which of course has a unique solution. We also have that $q(u_o) = [P(0)\, x\, (0),\, x\, (0)]$.

The main problem is then to show existence of solution of (the Riccati equation) (2.10). The main difficulty arises from the fact that C is unbounded, uncloseable, and is not treated in current works on the operator Riccati equation

[4,5]. Here we shall exploit the dual filtering problem for this purpose, which is also of interest on its own.

3. A Filtering Problem

Let us consider the stochastic system:

$$\dot{x}(t,\omega) = A^*x(t,\omega) + F\,\omega(t), \text{ a.e. } 0 < t < T;\ x(0) = 0 \tag{3.1}$$

where A is the same generator as in section 2, F is linear bounded, mapping $H_s \times H_b$ into H_b, such that $FF^* = Q^*Q$ and,

$$\omega(t) = \begin{matrix} \omega_1(t) \\ \omega_2(t) \end{matrix}$$

where $\omega_1(\cdot)$, $\omega_2(\cdot)$ are independent standard white Gaussian noises in W_s and W_b respectively, and hence $\omega(\cdot)$ defines similar white noise in $W_s \times W_b$. We have of course that

$$x(t,\omega) = \int_0^t S^*(t-\sigma)\ F\omega(\sigma)d\sigma$$

and for each ω, $x(t,\omega)$ is continuous in t.

The observation is

$$y(t,\omega) = C\ x\ (t,\omega) + \omega(t) \quad \text{a.e. } 0 < t < T \tag{3.2}$$

where

$$FG^* = 0$$

$$GG^* = \text{Identity}$$

We have 'distributed' state noise, and the 'observation' noise is on the boundary. The filtering problem for such a system when C is bounded (so that $Cx(t,\omega)$ is defined for every t) is treated in [1]. Here we consider a case

where C is unbounded, uncloseable; more specifically, we take C as defined in section 2. Then as we have seen, for each $\omega(\cdot)$,

$$x(t,\omega) \; \varepsilon \text{ Domain of } C \text{ a.e. } 0 < t < T.$$

so that (3.2) is well defined for each ω. Moreover

$$Cx(t,\omega) = D^* A^* x(t,\omega) = D^* \int_0^t A^* S^*(t-\sigma) F \omega(\sigma) d\sigma$$
$$= \int_0^t CS^*(t-\sigma) F \omega(\sigma) d\sigma \quad \text{a.e.} \quad 0 < t < T$$

and hence $Cx(\cdot,\omega)$ is an element of W_b and thus (3.2) can be looked upon as a mapping into W_b for each $\omega(\cdot)$ in $W_s \times W_b$ (Cartesian product).

Let us use the notation:

$$W_b(t) = L_2[[0,t]; L_2(\Gamma)]$$
$$W_s(t) = L_2[[0,t]; L_2(\mathscr{D})]$$

Then

$$y(s,\omega) \quad 0 < s < t$$

is in $W_b(t)$ and is a second order random variable therein. In fact denoting it by $\eta(t,\omega)$, we have:

$$E C\eta(t,\omega) \eta(t,\omega)^*] = I + (CL(t)) (CL(t))^*$$

where

$$L(t)f = g; \; g(s) = \int_0^s S^*(s-\sigma) F f(\sigma) d\sigma \quad 0 < s < t$$

$$CL(t)f = g; \; g(s) = \int_0^s CS^*(s-\sigma) F f(\sigma) d\sigma \quad \text{a.e.} \quad 0 < s < t$$

and $L(t)$ and $CL(t)$ define linear bounded transformations on $W_s(t) \otimes W_b(t)$ into $W_s(t)$ and $W_b(t)$ respectively. Hence we know that

$$\hat{x}(t,\omega) = E[\, x(t,\omega) \mid \eta(t,\omega)]$$

$$= M(t)\,(CL(t))^*\,[I + (CL(t)\,(CL(t))^*)^{-1}\,\eta(t,\omega) \qquad (3.3)$$

where

$$M(t)f = x;\ x = \int_0^t S^*(t-\sigma)F\, f(\sigma)d\sigma.$$

Since

$$C\, x(t,\omega)$$

is only defined a.e. in t, we <u>cannot</u> talk about $E(Cx(t,\omega) \mid \eta(t,\omega)]$ and we must instead consider

$$Cx(\cdot,\omega)$$

as an element of W_b. We have then

<u>Theorem 1</u>.

$$\hat{x}(t,\omega) \in \text{Domain of } A^* \quad \text{a.e.} \quad 0 < t < T$$

and

$$A^*\, \hat{x}(\cdot,\omega)$$

is an element of W_s for each ω, and defines a second order random variable in W_s.

<u>Proof</u>. We can clearly find a sequence of linear bounded operators T_n mapping H_s into the domain of A^* and such that T_n converges to the Identity strongly. Hence, for each t, $0 \leq t \leq T$:

$$E[A^*T_n x(t,\omega)\ |\eta(t)] = A^*T_n\hat{x}(t,\omega)$$

Moreover, we have, for each $h(\cdot)$ in W_s

$$E[A^*(T_n - T)\hat{x}(\cdot,\omega),h]^2$$

$$\leq E[A^*(T_n - T_m)\ x\ (\cdot,\omega),h]^2$$

Since for any f cn W_s,

$$E([\hat{x},f](x-\hat{x},f]) = E(\int_0^T [\hat{x}(t,\omega),f(t)] \int_0^t [x(s,\omega) - x\ (s,\omega),\ f(s)]dsdt) +$$
$$E(\int_0^T [x(s,\omega) - x\ (s,\omega),\ f(s)] \int_0^s [\hat{x}(t,\omega),\ f(t)]dt\ ds) = 0$$

But the random variables $[A^*T_n x(\cdot,\omega),h]$ converge to $[A^*x(\cdot,\omega),h]$ in the mean of order two. Hence

$$[A^*T_n\hat{x}(\cdot,\omega),h]$$

converges in the mean of order two. But we can write

$$[A^*T_n\hat{x}(\cdot,\omega),h] = (\omega,L_n h]$$

and $E[A^*T_n\hat{x}(\cdot,\omega),h]^2 = ||L_n h||^2$

Hence

$[\omega,L_n h]$ converges for each ω

Hence

$[A^*T_n\hat{x}(\cdot,\omega),h]$ converges for each ω to $[L\omega,h]$.

But since A^* is closed, it follows that

$$\hat{x}(t,\omega) \in \mathcal{D}[A^*] \quad \text{a.e.}$$

and of course

$$A^*\hat{x}(\cdot,\omega) = L\omega \in W_s.$$

and defines a second-order random variable. Hence

$$C\hat{x}(\cdot,\omega) = D^* A^*\hat{x}(\cdot,\omega)$$

defines a second order random variable in W_b also.

Again let $\mathscr{L}$ denote the operator such that

$$\hat{x}(\cdot,\omega) = \mathscr{L}y(\cdot,\omega)$$

where $\mathscr{L}$ is of course linear bounded, as can be verified from (3.3). Similarly let

$$x(\cdot,\omega) = L\omega$$

Then

$$A^*\hat{x}(\cdot,\omega) = A^*\mathscr{L}(G\omega+L\omega)$$

Given any element h in W_b we can find ω such that

$$G\omega + L\omega = h$$

Indeed we have only to take

$$\omega = G^*h$$

Hence $\mathscr{L}$ maps W_b into the domain of A^* and hence $A^*\mathscr{L}$ and hence also $C\mathscr{L}$ are both linear bounded.

Next we shall show that

$$z(\cdot,\omega) = y(\cdot,\omega) - C\hat{x}(\cdot,\omega)$$

is white noise in W_b. But this can be proved as in [1] using the fact that

$$E\left(\int_0^T [x(t,\omega) - C\hat{x}(t,\omega),h(t)] \int_0^t [y(s,\omega) - C\hat{x}(s,\omega),h(s)]ds\, dt\right) = 0$$

Let

$$E([x(t,\omega) - \hat{x}(t,\omega),h]\,[x(t,\omega) - \hat{x}(t,\omega),g]) = [P(t)h,g]$$

Our object is to show that $P(T-t)$ satisfies (2.10).

For this let us note that

$$y(\cdot,\omega) = (I - C\mathscr{L})^{-1}\, z(\cdot,\omega)$$

and

$$\hat{x}(\cdot,\omega) = \mathscr{L}\,(I-C\mathscr{L})^{-1}\, z(\cdot,\omega).$$

Let

$$(I - CS^*(1/n)\mathscr{L})^{-1} = I - M_n$$

where

M_n is Volterra and so is $\mathscr{L}(I-M_n)$,

and

$\mathscr{L}(I - M_n)$ converges strongly to $\mathscr{L}(I - C\mathscr{L})^{-1}$. Hence it follows that

$$\hat{x}(\cdot,\omega) = \lim_n \mathscr{L}(I - M_n)\, z(\cdot,\omega),\quad \hat{x}(t,\omega) = E[x(t,\omega)\,|\,\xi(t,\omega)] = F(t)\,\xi(t,\omega)$$

where

$$\xi(t,\omega) = z(s,\omega) \quad 0 < s < t$$

as an element of $W_b(t)$. Also

$$E[\, x(t,\omega) \quad \xi(t,\omega)^*] = F(t)$$

But for $h(\cdot)$ in W_b,

$$E([x(t,\omega),x] \int_0^t [z(\sigma,\omega),h(\sigma)]d\sigma)$$

$$= E[(x(t,\omega),x] \int_0^t [Cx(\sigma,\omega) - C\hat{x}(\sigma,\omega),\, h(\sigma)]d\sigma \tag{3.4}$$

Now

$$E([\, x(t,\omega),x]\, [\, x(\sigma,\omega) - \hat{x}(\sigma,\omega),\, y)])$$

$$= E[(S(t,\sigma)x(\sigma,\omega),x]\, [x(\sigma,\omega) - \hat{x}(\sigma,\omega),y]$$

$$= [P(\sigma)S(t-\sigma)x,y]$$

and hence (3.4)

$$= \int_0^t [CP(\sigma)S(t-\sigma)x,h(\sigma)]d\sigma \tag{3.5}$$

where

$$P(\sigma)S(t-\sigma)x \;\; \varepsilon \;\; \text{Domain of } A^* \;\; \text{a.e. } 0 < \sigma < t$$

and

$$\int_0^t ||A^*P(\sigma)S(t-\sigma)x||^2 d\sigma \quad < \infty$$

and hence also

$$\int_0^t ||D^*A^*P(\sigma)S(t-\sigma)x||^2 d\sigma < \infty$$

But (3.5)

$$= [x, F(t)h(\cdot)]$$

and

$$|[x, F(t)h]|^2 \leq [R(t,t)x, x] \int_0^t ||h(\sigma)||^2 d\sigma$$

where

$$R(t,t)x = E(x(t,\omega)x(t,\omega)^*]x = \int_0^t S^*(\sigma)FF^*S(\sigma)x d\sigma$$

Hence

$$||F(t)^*x||^2 \leq [R(t,t)x, x] \leq t||x||^2||FF^*|| M(t), [M(t) = \text{Sup } ||S(\sigma)|| \leq \sigma \leq t)$$

Hence

$$\left(\int_0^t ||CP(\sigma)S(t-\sigma)x||d\sigma\right)^2 \leq t^2||x||^2||FF^*|| M(t)$$

Using this, we shall show now that for x in the domain of A,

$$P(\sigma)x \in \mathcal{D}(A^*) \quad \text{a.e.}$$

Let λ be sufficiently large and positive so that it is in the resolvent set of A. Then: letting

$$C_n = CS^*(1/n)$$

we note that

$$\int_0^\infty e^{-\lambda t} \int_0^t (C_n-C_m)\ P(\sigma)\ S(t-\sigma)x d\sigma\ dt$$

$$= \int_0^\infty e^{-\lambda\sigma}(C_n-C_m))P(\sigma)R(\lambda,A)x d\sigma$$

where $R(\lambda,A)$ is the resolvent of A. Moreover

$$\int_0^\infty e^{-\lambda\sigma}||(C_n-C_m)P(\sigma)\ R(\lambda,A)x||d\sigma$$

$$\leq \int_0^\infty e^{-\lambda t} \int_0^t ||(C_n-C_m)\ P(\sigma)\ S(t-\sigma)\ x\ ||d\sigma\ \ dt \tag{3.6}$$

But

$$||(C_n-C_m)\ P(\sigma)\ S(t-\sigma)x||$$

$$= ||D^*(S^*(1/n) - S^*(1/m))A^*P(\sigma)S(t-\sigma)x||$$

and

$$||C_nP(\sigma)\ S(t-\sigma)x|| \leq ||D^*||\ ||A^*P(\sigma)S(t-\sigma)x||. \text{ (Constant)}$$

Hence the right side of (3.6) is Cauchy.

But

$$e^{-\lambda\sigma}S^*(1/n)P(\sigma)\ S(t-\sigma)x$$

$$\rightarrow e^{-\lambda\sigma}\ P(\sigma)S(t-\sigma)x$$

and

$$||e^{-\lambda\sigma}|S^*(1/n)P(\sigma)S(t-\sigma)x||$$

$$\leq ||e^{-\lambda\sigma}P(\sigma)S(t-\sigma)x||. \text{ (Constant)}$$

which is integrable on $(0,\infty)$. Hence it follows (since A^* is closed) that

$$P(\sigma)\ R(\lambda,A)x \ \ \varepsilon\ \mathscr{D}(A^*) \ \ \text{a.e.}$$

Hence for x in the domain of A,

$$P(\sigma)x \ \ \varepsilon\ \mathscr{D}(A^*) \ \ \text{a.e.},$$

and

$$\int_0^t ||A^*P(\sigma)x||d\sigma \ < \ \infty$$

for each $L > 0$.

In particular, since the semigroup is analytic, for every $t > 0$

$$S(t)x \ \ \varepsilon \text{ domain of } A^*P(\sigma) \ \ \text{a.e.}$$

Next we can readily see that

$$E[x(t,\omega),x]^2 \ = \ \int_0^t ||CP(\sigma)S(t-\sigma)x||^2 d\sigma$$

Hence

$$[P(t)x,y] \ = \ \int_0^t [S(\sigma)^*Q^*Q\ S(\sigma)x,y]d\sigma - \int_0^t [CP(\sigma)S(t-\sigma)x,\ CP(\sigma)S(t-\sigma)y]d\sigma$$

For x,y in the domain of A, it follows readily that, a.e.:

$$[\dot{P}(t)x,y] = [P(t)Ax,y] + [P(t)x,Ay] + \ [Qx,Qy] - [CP(t)x,\ CP(t)y]$$

More $P(T-t)$ then satisfies (2.10) as required.

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This paper solves the linear quadratic regulator problem for boundary control of partial differential equations using semigroup theoretic methods. The characteristic feature of this problem is that one has to allow for the operator involving the input to be unbounded, and in the corresponding 'dual' filtering problem, the observation operator (normal derivative on the boundary) has to be unbounded and unclosed.